Construction of coherent states for physical algebraic systems

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We construct a general state which is an eigenvector of the annihilation operator of the generalized Heisenberg algebra. We show, for several systems characterized by different energy spectra, that this general state satisfies the minimal set of conditions required to obtain Klauder's minimal coherent states.

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I. INTRODUCTION

Coherent states (CS's) were introduced by Schrödinger in 1926 [1] while he was studying the one-dimensional harmonic oscillator system. The same mathematical objects, the coherent states, were also studied by Glauber [2] and Klauder [3] four decades ago. Glauber found these states while he was studying the electromagnetic correlation function [2]. He also realized that these states have the interesting property of minimizing the Heisenberg uncertainty relation. Thus, one could say that these states are the quantum states with the behavior closest to a classical system. CS's have applications in many areas of physics [4], and since the birth of these states, there has always been some interest in investigating their algebraic properties [4,5].

We would like to point out that there is not one unique way to construct coherent states. In fact, there are a number of approaches—for instance, the well known Klauder [6] and Perelomov-Gilmore approaches [7]. In the first approach the coherent states are constructed using the basis of the Fock representation of the harmonic oscillator algebra, while in the second one this construction is based on notions of group theory.

In our work we deal with Klauder's approach, which is based on the construction of coherent states of the Heisenberg algebra. This algebra appears in many areas of modern theoretical physics and as an example we notice that the one-dimensional quantum oscillator algebra is an important tool in the second-quantization approach.

Due to the relevance of Heisenberg algebra, during the last two decades some effort has been devoted to studying possible deformations of the harmonic oscillator algebra [8]. During these years several groups have also generalized the Heisenberg algebra (see, for instance, Ref. [9–12]). [1] All these generalized Heisenberg algebras (GHA's) are related to each other. In this paper we will use the GHA given in Ref. [11] since, in this version of the algebra, the Hamiltonian of the physical system under consideration belongs explicitly to the set of generators of the algebra, the other generators in this set being the step operators of the system.

The version of the GHA given in Ref. [11] is written using a general function $f(x)$ called the characteristic function of the algebra, which is connected with the energy spectrum of the physical system under consideration. It was shown in Ref. [13] that there is a class of quantum systems described by this GHA. This class is characterized by those quantum systems having energy eigenvalues written as $\epsilon_{n+1}=f(\epsilon_n)$ where ϵ_n and ϵ_{n+1} are successive energy levels and $f(x)$ is a different function for each physical system.

Motivated by the procedure for constructing the standard coherent states of the harmonic oscillator, in this paper we build a state which is an eigenstate of the annihilation operator of the GHA having infinite-dimensional representations. In the main part of this paper we discuss the circumstances under which this general vector state, the eigenstate of the annihilation operator of the GHA, satisfies the minimum set of conditions required to construct Klauder's coherent states for the following systems: (i) harmonic oscillator, (ii) deformed harmonic oscillator, (iii) a general class of spectra, and (iv) free particle in a square well potential.

This paper is organized as follows: In Sec. II we summarize the GHA; in Sec. III we present a general expression for an eigenstate of the annihilation operator of the GHA and show that this expression satisfies the minimum set of conditions required to construct Klauder's coherent states for the cases enumerated above; in Sec. IV we present our conclusions.

II. GENERALIZED HEISENBERG ALGEBRA

Let us begin by reviewing the version of the GHA [9–12] given in Ref. [11]. We stress once more that all these generalized Heisenberg algebras are related to each other. The version of the GHA we are going to review is described by the generators $J_0, A, A^\dagger$, satisfying [11]

$$J_0 A^\dagger = A^\dagger f(J_0), \tag{1}$$

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[1] A generalized Heisenberg algebra is not necessarily a deformed Heisenberg algebra.

$$AJ_0 = f(J_0)A, \tag{2}$$

$$[A^\dagger, A] = J_0 - f(J_0), \tag{3}$$

where $A=(A^\dagger)^\dagger$, $J_0=J_0^\dagger$ is the Hamiltonian of the physical system under consideration, and $f(J_0)$ is an analytic function of J_0, called the characteristic function of the algebra. A large class of type Heisenberg algebras[2] can be obtained by choosing the appropriate function $f(J_0)$. It is interesting to note that in order to study generalized su(2) algebras we have to use a slightly different algebraic structure, as can be seen in Ref. [14]. The Casimir operator of the GHA has the expression

$$C = A^\dagger A - J_0 = AA^\dagger - f(J_0). \tag{4}$$

This algebra has a connection with the algebra independently proposed in Ref. [12], where the authors introduced the Heisenberg algebra through the set of elements (a^-, a^+, I), satisfying

$$[a^-, a^+] = a^-a^+ - a^+a^- \equiv \Delta', \tag{5}$$

$$[a^-, \Delta] = \Delta' a^-, \tag{6}$$

$$[\Delta, a^+] = a^+\Delta', \tag{7}$$

with $\Delta = a^+a^-$. The connection between Eqs. (1)–(3) and (5)–(7) can be made by means of the simple identification

$$\Delta' = f(J_0) - J_0, \tag{8}$$

$$a^+ = A^\dagger, \tag{9}$$

$$a^- = A, \tag{10}$$

$$a^+a^- = J_0, \tag{11}$$

$$a^-a^+ = f(J_0). \tag{12}$$

Before starting the construction of the coherent states associated with some physical systems by means of their related algebra, let us give a summary of its representation theory: the n-dimensional irreducible representations of the algebras (1)–(3) and (5)–(7) are given through the lowest eigenvalue of J_0 with respect to the vacuum state $|0\rangle$:

$$J_0|0\rangle = \alpha_0|0\rangle. \tag{13}$$

It is clear that for each value of α_0 and for a set of parameters of the algebra (related to the function f), we have a different vacuum, all of them denoted here, for simplicity, by $|0\rangle$. The solution of the representation theory problem is given in Ref. [11] for the linear and quadratic polynomials. The n-dimensional representation theory is given through a general vector $|m\rangle$ that is required to be an eigenvector of J_0,

$$J_0|m\rangle = \alpha_m|m\rangle, \tag{14}$$

where $\alpha_m = f^{(m)}(\alpha_0)$, the mth iterate of α_0 under f, and under the action of A and $A^\dagger$ we have

$$A^\dagger|m\rangle = N_m|m+1\rangle, \tag{15}$$

$$A|m\rangle = N_{m-1}|m-1\rangle, \tag{16}$$

where $N_m^2 = \alpha_{m+1} - \alpha_0$.

In Ref. [11] it was shown that choosing for the characteristic function of the GHA the linear function $f(x)=x+1$ the algebra in Eqs. (1)–(3) becomes the harmonic oscillator algebra and for $f(x)=qx+1$ we obtain in Eqs. (1)–(3) the deformed Heisenberg algebra. Moreover, it was shown in Ref. [13] that there is a class of quantum systems described by these generalized Heisenberg algebras. This class is characterized by those quantum systems having energy eigenvalues written as

$$\epsilon_{n+1} = f(\epsilon_n), \tag{17}$$

where ϵ_{n+1} and ϵ_n are successive energy levels and $f(x)$ is a different function for each physical system. This function $f(x)$ is exactly the same function that appears in the construction of the algebra in Eqs. (1)–(3), which was called the characteristic function of the algebra. In the algebraic description of this class of quantum systems, J_0 is the Hamiltonian operator of the system, and $A^\dagger$ and A are the creation and annihilation operators. This Hamiltonian and the ladder operators are related by Eq. (4) where C is the Casimir operator of the representation associated to the quantum system under consideration.

III. COHERENT STATES

Now, we are in a position to build the coherent states corresponding to some particular form of the characteristic function corresponding to the GHA with infinite-dimensional representations. Let us construct a state $|z\rangle$ which is an eigenstate of the annihilation operator of the GHA introduced in the previous section—i.e.,

$$A|z\rangle = z|z\rangle, \tag{18}$$

where z is a complex number. We expand the state $|z\rangle$ as $|z\rangle = \sum_{n=0}^{\infty} c_n|n\rangle$. Acting the annihilation operator of the GHA on $|z\rangle$ and using Eqs. (16) and (18) we have

$$A|z\rangle = \sum_{n=0}^{\infty} c_{n+1}N_n|n\rangle = z\sum_{n=0}^{\infty} c_n|n\rangle. \tag{19}$$

Equating the coefficients of $|n\rangle$ gives $c_{n+1}N_n = zc_n$. The solution of this equation for arbitrary c_n is

$$c_n = c_0\frac{z^n}{N_{n-1}!}, \tag{20}$$

where by definition $N_n! \equiv N_0N_1\cdots N_n$ and by consistency $N_{-1}! \equiv 1$. We will see in what follows that this definition of ! reduces to the standard definition of the factorial for the harmonic oscillator case. With the solution given in Eq. (20) we obtain, for the state $|z\rangle$,

$$|z\rangle = N(z)\sum_{n=0}^{\infty}\frac{z^n}{N_{n-1}!}|n\rangle, \tag{21}$$

where we have used $N(z)$ instead of c_0.

[2]A type Heisenberg algebra is an algebra having annihilation and creation operators among its generators.

It is worth mentioning that special characteristic functions will provide the GHA with finite-dimensional representations. We are going to consider here only characteristic functions generating the GHA with infinite-dimensional representations.

Let us now recall what the minimal set of conditions to obtain Klauder's coherent states (KCS's) are.

A state $|z\rangle$ is called a KCS if it satisfies the following conditions:

(i) normalizability

$$\langle z|z\rangle = 1; \tag{22}$$

(ii) continuity in the label

$$|z - z'| \to 0, \quad \| |z\rangle - |z'\rangle \| \to 0; \tag{23}$$

(iii) completeness

$$\int d^2 z w(z) |z\rangle\langle z| = 1. \tag{24}$$

We are going now to analyze the above minimal set of conditions to obtain a KCS for the state given in Eq. (21) in several examples.

A. Harmonic oscillator

As commented on in the previous section, the GHA reduces to the Heisenberg algebra by choosing the linear function $f(x)=x+1$ for its characteristic function. In this case we have $N_{n-1}^2=n$ and Eq. (21) becomes the standard coherent state for the harmonic oscillator with normalization coefficient given by $N \equiv N(z) = \exp(-|z|^2/2)$ and the weight function $w(r)$, $r=|z|$, required by the third condition is $w(r) = 1/\pi$.

B. Deformed Heisenberg algebra

As discussed in Ref. [11] by choosing the characteristic function of the GHA as $f(x)=qx+1$ we obtain a deformed Heisenberg algebra. In this case since $N_{n-1}^2 = N_0^2 [n]_q$, where $N_0^2 = \alpha_0(q-1)+1$, the Gauss number being $[n]_q=(q^n-1)/(q-1)$ and α_0 is the eigenvalue of the Casimir for the representation.

In the case we are analyzing Eq. (21) becomes

$$|z\rangle = \frac{N(|z|)}{N_0} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{[n]_q!}} |n\rangle, \tag{25}$$

where $[n]_q! \equiv [1]_q [2]_q \cdots [n]_q$ and $[0]_q! \equiv 1$. Using the normalizability condition we have

$$|z\rangle = \left[\sum_{n=0}^{\infty} \frac{|z|^{2n}}{[n]_q!} \right]^{-1/2} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{[n]_q!}} |n\rangle. \tag{26}$$

As discussed in Ref. [15] the function $g(z) = \sum_{n=0}^{\infty} |z|^{2n} / |n|_q!$ which appears in the above equation is convergent within a circle of radius $1/(1-q)$ for $0 < q < 1$ and outside this circle the function is defined by analytic continuation. For the classical case ($q=1$) it was shown that the completeness condition is achieved with $w(z)=1/\pi$ (see Ref. [15] for details on the construction of the weight function for this case).

C. Class of spectra

Let us now apply Eq. (21) to a simple class of spectra and then to the physically important case of the free particle in a square well potential. The key point is to know the analytical expression of the energy levels as shown below.

1. Spectrum type 1

Let us consider a system whose spectrum is given by the expression

$$\varepsilon_n = 1 - \frac{1}{n+1} = \frac{n}{n+1}, \quad \text{with } n \geq 0. \tag{27}$$

To obtain the characteristic function of the generalized algebra associated with this spectrum, we remark that

$$\varepsilon_{n+1} = \frac{n+1}{n+2} = \frac{1}{\dfrac{n}{n+1} + \dfrac{2}{n+1}}. \tag{28}$$

As

$$\varepsilon_n = \frac{n}{n+1} \text{ and } 1 - \varepsilon_n = \frac{1}{n+1}, \tag{29}$$

the substitution of Eq. (29) in Eq. (28) allows us to obtain the recurrence equation

$$\varepsilon_{n+1} = \frac{1}{2 - \varepsilon_n}. \tag{30}$$

Thus,

$$\varepsilon_{n+1} = f(\varepsilon_n) = \frac{1}{2 - \varepsilon_n}, \tag{31}$$

allowing us to identify the characteristic function f to be used in the algebra associated with this energy spectrum:

$$f(x) = \frac{1}{2 - x}. \tag{32}$$

As for the present spectrum

$$\alpha_n = \varepsilon_n = \frac{n}{n+1}, \quad \alpha_0 = 0, \tag{33}$$

it is thus easy to see that

$$N_{n-1}^2 = \frac{n}{n+1}, \tag{34}$$

yielding

$$N_{n-1}! = \frac{1}{(n+1)^{1/2}}. \tag{35}$$

The vector $|z\rangle$ defined in Eq. (21) can thus be written as

$$|z\rangle = N(|z|^2) \sum_{n \geq 0} (n+1)^{1/2} z^n |n\rangle. \tag{36}$$

Now, following our proposal concerning the definition of KCS's, we have to verify the three conditions mentioned above. Requiring that $\langle z|z\rangle = 1$ (normalizability condition) and remembering that $\langle m|n\rangle = \delta_{m,n}$, one obtains

$$\langle z|z\rangle = N^2(|z|^2)\sum_{m\geqslant 0}(m+1)|z|^{2m}. \qquad (37)$$

As

$$\sum_{m\geqslant 0}(m+1)|z|^{2m} = \frac{1}{(1-|z|^2)^2},$$

we have, for the normalization factor,

$$N^2(|z|^2) = (1-|z|^2)^2, \qquad (38)$$

where $0\leqslant |z| < 1$. Let us remark that with this result, obtained from a particular spectrum, the KCS can be constructed with a normalization function that is different from the exponential function which is the standard case. The second condition (continuity condition) is automatically verified. But to satisfy the third one, which is, in general, the most important, we have to find the weight function allowing the equality

$$\int d^2z w(|z|^2)|z\rangle\langle z| = 1. \qquad (39)$$

This expression means the overcompleteness condition in the KCS domain for the particular case of the spectrum of type 1. Substituting Eqs. (36) and (38) in Eq. (39) and integrating on the angle θ [$z=r\exp(i\theta)$], we obtain the expression

$$2\pi\sum_{m\geqslant 0}|m\rangle\langle m|\int_0^1 dr w(r^2)N^2(r^2)(m+1)r^{2m+1}. \qquad (40)$$

Changing $x=r^2$, this expression can be written as

$$\pi\sum_{m\geqslant 0}|m\rangle\langle m|\int_0^1 dx w(x)N^2(x)(m+1)x^m, \qquad (41)$$

and remarking that

$$\int_0^1 dx(n+1)x^n = 1, \qquad (42)$$

it is obvious that we can solve Eq. (39) if we choose the weight function w satisfying the condition

$$\pi w(x)N^2(x) = 1. \qquad (43)$$

The explicit form of w, allowing the resolution of the completeness equation can, finally, be written as

$$w(x) = \frac{1}{\pi(1-x)^2}. \qquad (44)$$

2. Spectrum type 2

Now, we are going to treat the case of the quadratic spectrum in this class of spectra. Let us call quadratic spectrum the spectrum ε_m having the following expression:

$$\varepsilon_m = \left(1-\frac{1}{m+1}\right)^2 = \frac{m^2}{(m+1)^2}, \qquad (45)$$

where $m=0,1,2,3,\ldots$. As in the previous case, we are interested in computing the characteristic function of the GHA for this particular spectrum. From the above expression, we have

$$\sqrt{\varepsilon_m} - 1 = \frac{-1}{m+1}, \qquad (46)$$

leading us to the expression

$$\varepsilon_{m+1} = \frac{(m+1)^2}{(m+2)^2} = \left(\frac{1}{2-\sqrt{\epsilon_m}}\right)^2. \qquad (47)$$

Consequently, the characteristic equation is given by

$$\varepsilon_{m+1} = f(\varepsilon_m), \qquad (48)$$

with

$$f(x) = \left(\frac{1}{2-\sqrt{x}}\right)^2. \qquad (49)$$

As before, let us consider Eq. (21). As $\alpha_0=\varepsilon_0=0$ and $N_{m-1}^2 = \alpha_m - \alpha_0 = \alpha_m = \varepsilon_m = [m/(m+1)]^2$, after some calculation, the scalar $\langle z|z\rangle$ can be written as

$$\langle z|z\rangle = N^2(|z|^2)\sum_{m\geqslant 0}(m+1)^2|z|^{2m}. \qquad (50)$$

The sum can be easily performed and we obtain

$$N^2(|z|^2) = \frac{(1-|z|^2)^3}{1+|z|^2}, \qquad (51)$$

where $0\leqslant|z|<1$. We note that once more the normalized function is not an exponential one.

As mentioned before the most important equation is the resolution of the completeness equation. To get this, we must find an adequate weight function w. Performing a computation similar as in previous cases, the weight function must obey ($x=|z|^2$)

$$\pi\sum_{m\geqslant 0}|m\rangle\langle m|\int_0^1 dx w(x)N^2(x)(m+1)^2x^m = 1. \qquad (52)$$

One can verify that a solution of this equation is given by

$$w(x) = -\frac{\ln x}{\pi N^2(x)}. \qquad (53)$$

Using Eq. (51), we can write the weight function as

$$w(x) = -\frac{\ln x}{\pi}\frac{1+x}{(1-x)^3}. \qquad (54)$$

This function ensures the resolution of the completeness equation, corresponding to the case of the spectrum of type 2, allowing the construction of coherent states for this kind of spectrum.

3. General case

The spectra of types 1 and 2 can be generalized to an arbitrary order. Let us now consider the general spectrum

$$\varepsilon_n = \left(1 - \frac{1}{n+1}\right)^{\alpha}, \qquad (55)$$

with $\alpha \geqslant 2$. The question now is to find the corresponding GHA. After that, we have to find the characteristic function. Starting from the fact that

$$\varepsilon_n^{1/\alpha} - 1 = -\frac{1}{n+1}, \qquad (56)$$

one can check that

$$\varepsilon_{n+1} = \left(\frac{n+1}{n+2}\right)^{\alpha} = \left(\frac{1}{1-\varepsilon_n^{1/\alpha}}\right)^{\alpha}. \qquad (57)$$

Then the characteristic function is

$$f(x) = \left(\frac{1}{2 - x^{1/\alpha}}\right)^{\alpha}. \qquad (58)$$

Let us now verify the minimal set of conditions for the state in Eq. (21) in the case of the general spectrum under consideration. For this general case we have

$$N_{n-1}\,! = \frac{1}{(n+1)^{\alpha/2}}, \qquad (59)$$

allowing us to write Eq. (21) in this case as

$$|z\rangle = N(|z|^2)\sum_{n\geqslant 0}(n+1)^{\alpha/2} z^n |n\rangle. \qquad (60)$$

As

$$\sum_{n\geqslant 0}(n+1)^{\alpha}|z|^{2n} = \frac{\mathrm{Li}_{-\alpha}(|z|^2)}{|z|^2}, \qquad (61)$$

where $\mathrm{Li}_{-\alpha}(|z|^2)$ is the polylogarithm function, the coefficient $N^2(|z|^2)$ can be written as

$$N^2(|z|^2) = \frac{|z|^2}{\mathrm{Li}_{-\alpha}(|z|^2)}, \qquad (62)$$

and $0 \leqslant |z| < 1$. The expression of the weight function that is behind the resolution of the unity equation is, nevertheless, harder to be obtained. Following the method used before, we find that, for the general case, the weight function can be written as $(x=|z|^2)$

$$w(x) = (-1)^{\alpha+1}\frac{(\ln x)^{\alpha-1}}{\pi\Gamma(\alpha)N^2(x)}, \qquad (63)$$

where $\Gamma(\alpha)$ is the gamma function. As an example, we consider the behavior of $w(x)$ for $\alpha=3$:

$$w(x) = \frac{(\ln x)^2}{2\pi}\frac{1+4x+x^2}{(1-x)^4}, \qquad (64)$$

which is shown in Fig. 1.

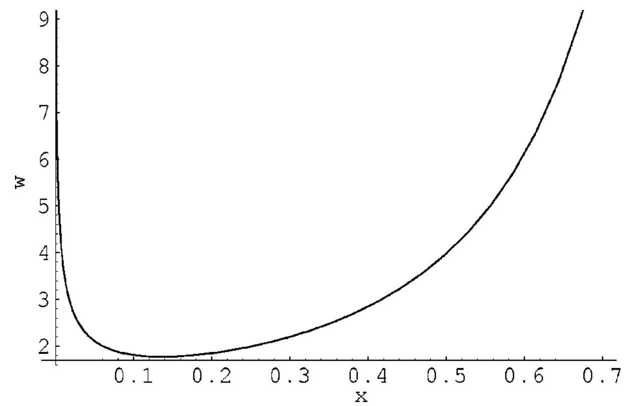

FIG. 1. Weight function for the system having characteristic function given as $f(x)=[1/(2-x)^{1/3}]^{1/3}$.

D. Free particle in a square-well potential

We are now going to compute the coherent states of a physical system using the formalism described before. The results in this case are more complicated because they involve a spectrum needing a weight function which is a special function. In fact, the latter is relatively not obvious in comparison with the ones introduced in the previous sections. Let us begin with the well-known spectrum of a free particle in a square-well potential:

$$\varepsilon_n = (n+1)^2, \quad n = 0,1,2,3,\ldots\ . \qquad (65)$$

Then,

$$\varepsilon_{n+1} = (n+2)^2 = \varepsilon_n + 2\sqrt{\varepsilon_n} + 1. \qquad (66)$$

Using the algebraic formalism shown before and observing that $(\alpha_n = \varepsilon_n)$

$$N_{n-1}^2 = \alpha_n - \alpha_0 = \alpha_n - 1 = n^2 + 2n, \qquad (67)$$

we obtain

$$N_{n-1}\,! = \frac{1}{\sqrt{2}}\sqrt{n!}\sqrt{(n+2)!}\,. \qquad (68)$$

We thus obtain for our proposal of coherent states given in Eq. (21) the expression

$$|z\rangle = \sqrt{2}N(|z|)\sum_{n\geqslant 0}\frac{z^n}{\sqrt{n!}\sqrt{(n+2)!}}|n\rangle. \qquad (69)$$

The normalizability condition can be fulfilled if we satisfy the expression

$$2N^2(|z|)\sum_{n\geqslant 0}\frac{|z|^{2n}}{n\,!\,(n+2)!} = 1. \qquad (70)$$

Noting that

$$\sum_{n\geqslant 0}\frac{|z|^{2n}}{n\,!\,(n+2)!} = \frac{I_2(2|z|)}{|z|^2}, \qquad (71)$$

for $0 \leqslant |z| < 1$, where $I_n(z)$ is the modified Bessel function of the first kind of order n, the expression for the normalizability coefficient can be written as

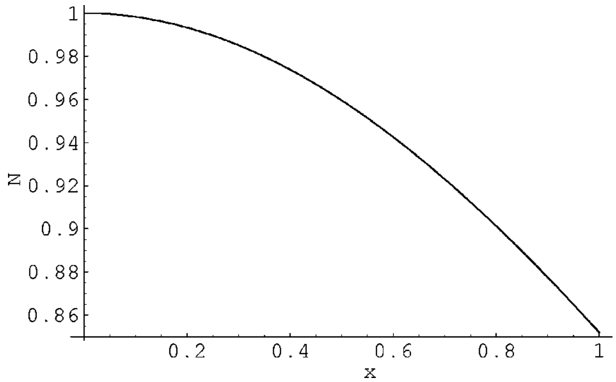

FIG. 2. Normalization function for the free particle in a square-well potential.

$$N^2(|z|) = \frac{|z|^2}{2I_2(2|z|)}, \tag{72}$$

where $0 \leqslant |z| < \infty$. The behavior of this function can be seen in Fig. 2.

The resolution of the completeness problem is given by finding the adequate weight function $w(x)$, $x=|z|^2$, satisfying the equality

$$\pi \sum_{n\geqslant 0} |n\rangle\langle n| \frac{2}{n\,!\,(n+2)!} \int_0^\infty dx \frac{w(\sqrt{x})N^2(\sqrt{x})x^{n+1}}{2x} = 1. \tag{73}$$

If we take

$$\frac{\pi w(\sqrt{x})N^2(\sqrt{x})}{2x} = K_2(2\sqrt{x}), \tag{74}$$

where $K_n(x)$ is the modified Bessel function of the second kind of order n, the weight function takes the form

$$w(\sqrt{x}) = \frac{2}{\pi} \frac{xK_2(2\sqrt{x})}{N^2(\sqrt{x})} \tag{75}$$

and can, finally, be written as

$$w(x) = \frac{4}{\pi} K_2(2\sqrt{x}) I_2(2\sqrt{x}), \tag{76}$$

which is shown in Fig. 3.

With this expression, one can verify that the important condition, the completeness equation, is satisfied by considering that

$$\int_0^\infty dx K_2(2\sqrt{x}) x^{n+1} = \frac{1}{2} n\,!\,(n+1)\,!\,. \tag{77}$$

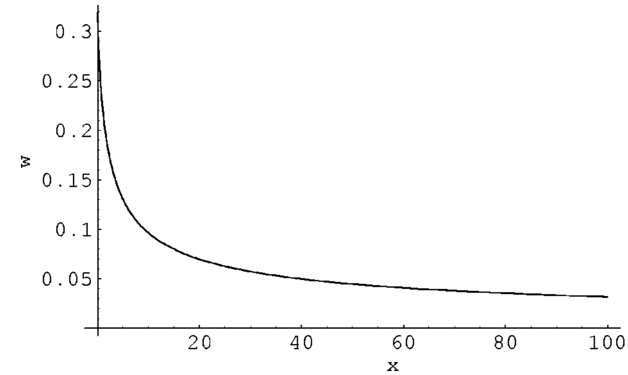

FIG. 3. Weight function for the free particle in a square-well potential.

IV. CONCLUSION

We have investigated in this work a state constructed as an eigenstate of the annihilation operator of the generalized Heisenberg algebra. We have shown for several systems (harmonic oscillator, deformed harmonic oscillator, a class of spectra, and the square-well potential) that this state satisfies the minimum set of conditions required to obtain Klauder's coherent states.

The GHA we considered is an algebra having as generators the Hamiltonian of the physical system under consideration and the annihilation and creation operators of the system. The state we have investigated is an eigenstate of the annihilation operator of the GHA for a general system described by this GHA. Thus, this state is a natural generalization for a general system described by the GHA of the coherent states of the standard harmonic oscillator system.

It is interesting to note that in the proof [Eqs. (18)–(21)] of our expression for coherent states given in Eq. (21) it was only necessary to admit (i) an infinite sum, (ii) $A|0\rangle=0$, and (iii) $A|n\rangle=N_{n-1}|n-1\rangle$. We see that this formalism is not applied to finite-dimensional representations of GHA's and that the explicit expression of N_n was not necessary in order to get Eq. (21). The explicit expression of N_n was necessary only when we showed, for specific spectra, that the state satisfied the minimal set of conditions to obtain Klauder's coherent states. Thus, we think that the expression in Eq. (21) could be a consistent definition of coherent states even for systems which are not described by the GHA but satisfying the conditions (i)–(iii) mentioned above.

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