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## Coherent States via Decoherence

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The rate at which pure initial states deteriorate into mixtures is computed for a harmonic oscillator interacting with an environment in thermal equilibrium. The decoherence process resulting from this interaction selects a set of states characterized by maximal stability (or minimal loss of predictive power) which can be quantified by the rate of increase in either linear or statistical entropy. In the weak coupling limit, coherent states are shown to produce the least entropy, thus becoming the natural counterparts of classical points in phase space.

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Coherent states were introduced into physics primarily because of mathematical convenience [1,2]. Later, it was noticed that these states have several unusual properties which allow them to play a special role whenever a harmonic motion is a good approximation. For example, in quantum optics their role is tied to mathematics, but also to physics, as they are naturally generated by a laser and prepared by a typical means of measurement (such as a photomultiplier [3]). Here we shall show that coherent states have one more remarkable property: When the harmonic oscillator is weakly coupled to a canonical (oscillator-heat bath) environment, coherent states are least susceptible to the loss of quantum coherence.

Schrödinger [1] suggested very early on that coherent states are the quantum counterparts of classical points in phase space. His argument was based on the stability and invariance of a Gaussian wave packet in an isolated quantum harmonic oscillator. It preceded the realization of many of the difficulties encountered in the interpretation of quantum mechanics. The hope that analogous counterparts of classical trajectories could be found for a general quantum system soon disappeared, in part as a result of persistent questioning by Bohr [4]. When the true nature of the difficulties posed by the conflict between the quantum superposition principle and its apparent violations in the “real world” (in particular, in the course of the quantum measurement process) became apparent [5], this naive first attempt proved to be clearly inadequate.

Recent discussions of the transition between the quan-

tum substrate and the familiar classical reality have focused on the “openness” of macroscopic quantum systems [6], the stability of various quantum states in the presence of an environment [7–9], and the resulting process of decoherence [9–13]. The question addressed by these considerations goes back at least to Einstein, who (in a letter to Born), inquired about the apparent absence of the majority of quantum states admissible in the Hilbert space from familiar, everyday experience [14].

The answer to such questions emerges naturally once the role played by the environment in the transition from quantum to classical is acknowledged: Macroscopic quantum objects are nearly impossible to isolate from their surroundings. It was shown for various idealized models (invented primarily to study the “reduction of the state vector” in the context of idealized measurements) that, when the environment—that is, degrees of freedom which interact with the record-keeping “pointer” of a quantum apparatus—is taken into account, the vast majority of pure states become in effect inaccessible [7–9]. This is because a continuous interaction with the environment destroys, on a very short *decoherence time scale*, the purity of nearly all of the initial superpositions. Thus, only the observations which refer to a preferred set of stable states or the associated set of observables will exhibit one of the key attributes of “classical reality,”—the predictive power of the associated records.

Such an interaction can be thought of as a continuous monitoring of the macroscopic quantum system by the environment. Neglecting the self-Hamiltonian of the sys-

tem, the eigenstates of the interaction Hamiltonian are natural *pointer states* [7] of the quantum apparatus, as they do not evolve at all. At the same time their superpositions can be shown to decay rapidly, regardless of the initial state of the apparatus, into nearly exact mixtures of pointer states. Thus, the dynamics of the complete system prevents the stable existence of states corresponding to superpositions of pointer positions and, therefore, results in an effective *environment-induced superselection rule* [7-9] which dynamically makes all such superpositions inaccessible.

Special assumptions, such as the irrelevance of the self-Hamiltonian, which are justifiable in idealized apparatus models, do not apply to other macroscopic systems which are also expected to behave classically. But the basic ideas regarding the openness of macroscopic quantum systems and the function of the environment in the apparent violation of the superposition principle on the classical level can still be regarded as a guide in the search for quantum causes of the emergent classicality [7-10]. A natural generalization of the absolutely stable pointer states of an apparatus in a situation where no pure state is absolutely stable would be the set of states which are least prone to deteriorate into a mixture. Since entropy is a good measure of the loss of purity and, at the same time, of the loss of predictability, a convenient criterion for states of an evolving open system is based on the increase of entropy of the density matrices resulting from various initial states. The relatively least unstable candidates for the set of preferred states are obviously those which yield the least entropy increase. This procedure has been referred to as the *predictability sieve* [10].

The search for quantum states which are closest to the classical "points in phase space" provides an alternative way of viewing the motivation for our study. Such states should define the location in phase space with optimal accuracy and evolve almost reversibly on a dynamical time scale in spite of being coupled to an environment. By looking at these states one is paying attention to a "high resolution" regime (in contrast to the low resolution  $\Delta x \Delta p \gg \hbar$  which is sometimes associated with the classical limit). While complete reversibility cannot be attained in an open system, it is still natural to ask what set of states is closest to this Newtonian ideal especially in the limit when the motion is nearly reversible. As the degree of irreversibility is quantified by the increase of entropy, we are again led to look for the states which result in least entropy production.

We will apply the predictability sieve to a harmonic oscillator undergoing quantum Brownian motion [11]. To estimate the entropy produced from some initial state we will utilize the master equation for the reduced density matrix. To simplify the analytic aspect of our presentation we will restrict ourselves to the case of weak coupling and high temperature. However, as we will show later, our results can be extended to the low-temperature re-

gime. The high-temperature master equation is

$$\dot{\rho} = -\frac{i}{\hbar} [H_R, \rho] - \frac{i\gamma}{2\hbar} [\{p, x\}, \rho] - \frac{2m\gamma k_B T}{\hbar^2} [x, [x, \rho]] - \frac{i\gamma}{\hbar} ([x, \rho p] - [p, \rho x]), \quad (1)$$

where  $[, ]$  and  $\{, \}$  are commutators and anticommutators, respectively, while  $H_R$  is the renormalized self-Hamiltonian of the system and  $\gamma$  is the relaxation rate. The above equation was derived for a system coupled linearly through its coordinate  $x$  to a heat bath of harmonic oscillators [11].

When Planck's constant is small relative to the actions involved and when the object is massive—conditions expected in the classical limit—the third term of (1) is dominant. It has been argued before by one of us that this allows for a reversible classical limit in which the nonlocal states are destroyed by the dissipative coupling with the environment, and yet states which are localized follow an approximately reversible equation of motion [9].

A computationally convenient way to implement the predictability sieve is to calculate the linear entropy production. The linear entropy,

$$s(\rho) = \text{Tr}(\rho - \rho^2), \quad (2)$$

is a good measure of the purity of a quantum state. When the interaction is weak ( $\gamma \sim 0$ ) and the state remains approximately pure we can use (1) to show that

$$\dot{s} = 4D \text{Tr}(\rho^2 x^2 - \rho x \rho x) - 2\gamma \text{Tr} \rho^2 \quad (3)$$

$$\simeq 4D(\langle x^2 \rangle - \langle x \rangle^2) = 4D\Delta x^2, \quad (4)$$

where  $D = 2\gamma m k_B T / \hbar^2$ . Thus the rate of linear entropy increase is proportional to the dispersion in position. This special role of position can be traced back to the form of the interaction between the system and the environment, which depends on and therefore commutes with  $x$ . In the absence of the self-Hamiltonian, this would make position eigenstates natural candidates for the pointer states. However, we will show here that due to the presence of the self-Hamiltonian, the pointer states turn out to be coherent states which are as "close" to momentum as they are to position eigenstates.

In the weak-coupling limit, we can integrate (4) replacing the free Heisenberg equations for the oscillator operators in the right-hand side. In this way, since position and momentum interchange their roles in the course of every oscillator period  $\tau$ , we obtain

$$s(\tau) = 4D \int_0^\tau dt \langle \psi | \left[ (x - \langle x \rangle) \cos \omega t + \frac{1}{m\omega} (p - \langle p \rangle) \sin \omega t \right]^2 | \psi \rangle = 2D \left[ \Delta x^2 + \frac{\Delta p^2}{m^2 \omega^2} \right], \quad (5)$$

where the initial dispersions in  $x$  and  $p$  appear on the right-hand side.

We are now in a position to use this estimate of the produced linear entropy together with data about the possible pure initial states to find the set of preferred, most predictable states. Considering the linear entropy (5) as a function of  $\Delta x$  and of the product  $\Delta x \Delta p$  (which, due to the uncertainty relations, is bounded by  $\hbar/2$ ) we find that *the minimum entropy-producing initial state is characterized by  $\Delta x \Delta p = \hbar/2$  (minimum uncertainty) and*

$$\Delta x^2 = \hbar/2m\omega, \tag{6}$$

which is of course the spread in position of the ground state or of a coherent state of the harmonic oscillator. This establishes the key result of our paper. Below, we shall compute explicitly the entropy production of squeezed states as a function of the initial squeeze parameter to show that our result is robust and can be extended to low temperatures.

A wave packet which is initially Gaussian remains Gaussian in the course of the quantum Brownian motion and its Wigner distribution function evolves into

$$W_G(x,p;t) = \frac{1}{2\pi} \frac{1}{\delta_x \delta_p} \exp \left\{ -\frac{(x - \bar{x}_t)^2}{2\delta_x(t)^2} - \frac{[p - \bar{p}_t + \beta(x - \bar{x}_t)]^2}{2\delta_p(t)^2} \right\}, \tag{7}$$

where  $\bar{x}_t$  and  $\bar{p}_t$  give the average time-dependent location of the system along its trajectory in phase space. The area covered by the wave packet (strictly speaking by its  $1-\sigma$  "isodensity contour") is related to the linear entropy through

$$s(\rho) = 1 - \frac{1}{A} = 1 - \frac{\hbar}{2\delta_x(t)\delta_p(t)}. \tag{8}$$

The "area"  $A$  in the two-dimensional phase space is given in units of Planck's constant. For a Gaussian mixed state the usual statistical entropy is also a simple function of  $A$ :

$$S(\rho) = \frac{A+1}{2} \ln(A+1) - \frac{A-1}{2} \ln(A-1) - \ln 2. \tag{9}$$

Both linear and statistical entropy are increasing functions of  $A$ , which can be computed as a function of time and has the following simple overall form (see Ref. [13] for technical details):

$$A = \{A_0^2 + A_0[sf_+(t) + s^{-1}f_-(t)] + h(t)\}^{1/2}, \tag{10}$$

where the initial area  $A_0 = 2\Delta x \Delta p / \hbar$  and the squeeze parameter  $s = m\omega \Delta x / \Delta p$  parametrize the initial state, while  $f_{\pm}$  and  $h$  are complicated functions of time and temperature.

To verify our key result we compute, for an initially pure ( $A_0 = 1$ ) state, the entropy at an arbitrary time and minimize its value, or equivalently that of the area in

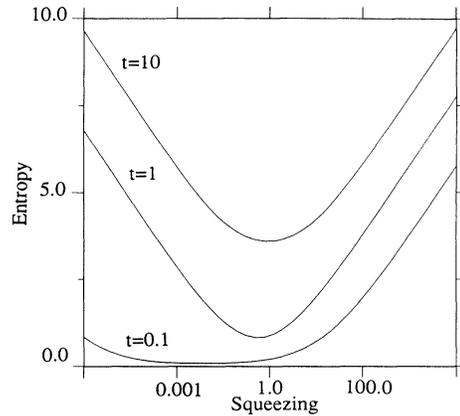


FIG. 1. The entropy-squeeze relationship as a function of time. The temperature,  $T=10^4$ , and the relaxation rate  $\gamma=10^{-4}$ . Time is measured in units of  $1/\omega$ . Note that at early times small values of the squeeze (corresponding to approximately position eigenstates) are preferred.

phase space, with respect to the initial state. This is accomplished when

$$s^2 = s_{\min}^2 = \frac{f_+(t)}{f_-(t)}. \tag{11}$$

The squeeze  $s_{\min}$  for minimum entropy production always exists as a function of time but the predictability sieve requires  $s_{\min}$  to be time independent over a range of dynamical times. In the high-temperature regime it is possible to obtain

$$f_{\pm}(t) \approx \frac{\gamma}{\omega} \frac{k_B T}{\hbar \omega} \left[ \omega t \mp \frac{1}{2} \sin 2\omega t \right], \tag{12}$$

which implies

$$s_{\min}^2 = \frac{2\omega t - \sin 2\omega t}{2\omega t + \sin 2\omega t}. \tag{13}$$

Therefore, for times of the order of the oscillator period

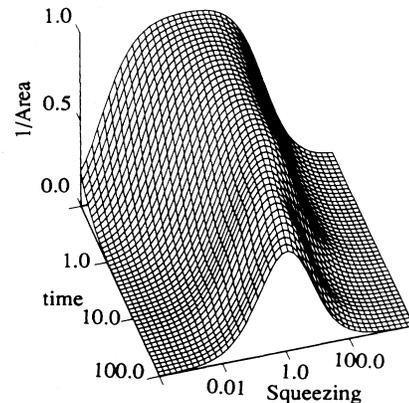


FIG. 2. The inverse area as a function of both time and the squeeze. Parameter values are those of Fig. 1. Coherent states are clearly picked out at late times.

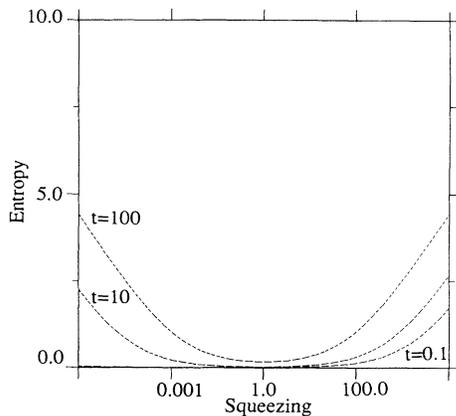


FIG. 3. As in Fig. 1, but with  $T=0$ . Coherent states are still preferred but decoherence now takes place over a longer time scale.

and greater, the least perturbed initial state will be a coherent state ( $s_{\min}=1$ ; see Figs. 1 and 2). In the low-temperature regime it is not possible to find simple expressions for  $f_{\pm}$  but these functions can be computed numerically. The result, plotted in Fig. 3, confirms the main result of our paper.

It is also of interest to extend this analysis by considering initial states that are not pure but mixed, exploring in this way the imperfect resolution (i.e.,  $\Delta x \Delta p > \hbar$ ) regime. Using Eqs. (10) and (12) we can show that, in the high-temperature regime and for  $\omega t \geq 1$ , the minimum entropy-producing states are also such that  $s_{\min}^2=1$  (which implies that  $\Delta p = m\omega\Delta x$ ). Thus, the minimum rate of increase of the area in phase space

$$\dot{A} = \gamma \frac{k_B T}{\hbar \omega} + O(\gamma^2) \quad (14)$$

is independent of the initial value of  $\Delta x \Delta p$ . However, the rates of increase of both the linear and statistical entropy (which can be used to quantify the rate of predictability loss) decrease with the increase of  $A_0$ . This is in accord with the expectation that stability and reversibility can be enhanced by giving up resolution.

We conclude by reiterating the importance of the main result for the understanding of the transition from quantum to classical: The predictability sieve naturally selects coherent states in the course of harmonic motion. These states, being pure, allow for maximal resolution in phase space. They can be regarded as the closest quantum counterparts of classical points.

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