PLA-D-14-01427

Title: A unified scheme of correlations using linear relative entropy

List of changes

We would like to thank the referees for their constructive remarks.
We did as best as we could to take into account all their comments.

List changes/ Reviewer 1

concerning the comment

1) Page 2, Introduction, second paragraph. At the end of the first sentence of the second paragraph "Therefore, quantification [...] last two decades [Refs]", the authors should add some relevant and general references about the characterization of quantum correlations both theoretical and experimental, in order to make their reference list more up-to-date. For instance, they should cite:
- J.-S. Xu, K. Sun, C.-F. Li, X.-Y. Xu, G.-C. Guo, E. Andersson, R. Lo Franco and G. Compagno, Nat. Commun. 4, 2851 (2013)
- R. Lo Franco, B. Bellomo, S. Maniscalco, and G. Compagno, Int. J. Mod. Phys. B 27, 1345053 (2013)
- I. A. Silva, D. Girolami, R. Auccaise, R. S. Sarthour, I. S. Oliveira, T. J. Bonagamba, E. R. deAzevedo, D. O. Soares-Pinto, and G. Adesso, Phys. Rev. Lett. 110, 140501 (2013)

our reply

We added the following sentence:
"The experimental and theoretical efforts, deployed in this context, are essential to develop the appropriate strategies to prevent against the de-coherence effects induced by the system-environment coupling (see for instance the recent works [13,14,15] and references therein."
quoting the suggested references.

concerning the comment

2) Page 2, Introduction, third line of second paragraph. Besides references [13-16], the authors should add some other relevant papers regarding the different measures of quantum correlations. For instance, they could cite:
- B. Aaronson, R. Lo Franco, and G. Adesso, Phys. Rev. A 88, 012120 (2013)
- Paula F M, Montealegre J D, Saguia A, de Oliveira T R and Sarandy M S (2013) Europhys. Lett. 103, 50008

- T. R. Bromley, M. Cianciaruso, R. Lo Franco, and Gerardo Adesso, arXiv:1404.1409.

our reply

We added the suggested references.

concerning the comment

3) Page 2, Introduction, line 3 of the third paragraph. The adjective "semi-classical" is not a very usual term for the correlations in the community. I believe they mean the kind of correlations that are quantum only with respect to one part of the system (connected to the quantum-classical states). Therefore, I suggest the authors to make citations here to papers where this term is explicitly used or to briefly explain this expression, maybe inside brackets.

our reply

Clearly the term semi-classical means correlations that are quantum only with respect to one component of the system.

We added the following comment :

(related to quantum-classical states) after the term semi-classical.

concerning the comment

4) Page 3, Introduction, line 4 from the top. The authors should change "mutual" with "total", since the correlations are "total" not "mutual". The mutual information characterize the total correlations.

our reply

This is corrected.

concerning the comment

5) Page 3, line 4 of the last paragraph of Introduction. At the end of the sentence: "On the other hand, the symmetric part is exactly the Hilbert-Schmidt distance", the authors should cite Ref. [35], where this fact has been first derived.

our reply

The suggested reference is cited.

concerning the comment

6) Page 3, third last line of the last paragraph of the introduction. When the authors write about "the classical state and its closest product state", it is not very clear that also the classical state must be the closest one to the system state. I suggest the authors to change the sentence like: "In this situation the explicit derivations of the suitable closest product and classical states is achieved".

our reply

This is adjusted.

concerning the comment

7) The title of section 2 should be corrected by adding "on" as: "Correlation quantifiers based on symmetrized linear relative entropy". This must be analogously done for the other subsection titles.

our reply

This is added.

concerning the comment

8) Page 3, first line after Eq. (1). The authors should correct "... given quantum..." with "... given quantumness...".

our reply

This is corrected.

concerning the comment

9) Page 4, first line. Please, correct "... densities matrices...." with "... density matrices...".

our reply

This is corrected.

concerning the comment

10) Page 4, second line from the top. Please correct "... von Neumann [34]" with "... von Neumann entropies [34]".

our reply

This is corrected.

concerning the comment

11) Page 4, the line before Eq. (4). At the end of the sentence the authors should explicitly write that χ_ρ is the closest classical state to ρ, so that the subsequent quantum and classical correlations are well defined.

our reply

Before the equation 4, we added : $\cdots$ the state ρ and its closest classical state

concerning the comment

12) Page 4, first line after E. (4). At the end of this line the authors must correct "... distance between the closest classical state χ_ρ ...".

our reply

This is corrected.

concerning the comment

13) Page 4, first line after Eq. (5). The authors should correct "... based relative entropy ..." with "relative entropy-based ...". They should check it everywhere in the paper.

our reply

This is corrected.

concerning the comment

14) Page 4, last line before Eq. (8). At the end of this sentence the authors should cite Ref. [35], where this linearization first appears. Moreover, they should correct Eq. (8) by changing the first ρ_1 with ρ_2, according to Eq. (11) of Ref. [35].

our reply

This is corrected.

concerning the comment

15) Page 5, Eq. (12). In this equation S_+ should appear instead of S_-. There is confusion here, since the author make reference to Eq. (9) and then write S_-.

our reply

Indeed, there we should make reference to S_-. This is corrected.

concerning the comment

16) Page 5, first line after Eq. (13). I am not convinced that Eq. (13), containing the linear entropy S_2, is a symmetrized form of the relative entropy, that contains logarithms. Maybe the authors mean "a linearized version of relative entropy". They should clarify this point.

our reply

after equation (13), we added "a symmetrized form of the linear relative entropy"

concerning the comment

17) Page 5, last line before Eq. (14). At the end of the sentence "... the Hilbert-Schmidt distance ", the authors should cite Ref. [35], where this aspect has been first pointed out.

our reply

We quote the suggested reference.

concerning the comment

18) Page 5, second line after Eq. (14). The authors should change "symmetrized' with "symmetric". In the successive line they should also change "antisymmetrical" with "antisymmetric", in order to be consistent with the definitions above.

our reply

This is corrected.

concerning the comment

19) Page 6, second line above Eq. (20). As usual, the authors should change " based linear relative entropy correlations" in "liner relative entropy based correlations".

our reply

This is corrected.

concerning the comment

20) Page 7, first line after Eq. (26). The condition on the coefficients c_1, c_2 to be comprised between 0 and 1 should be anticipated when these coefficients appear for the first time, that is together with the condition $c_1 + c_2 <= 1$ in the second line after Eq. (24). In fact, the state must be well-defined in both the representations (matrix and Fano-Bloch).

our reply

The condition $c_1 + c_2 \leq 1$ is now given after equation (26).

concerning the comment

21) Page 8, Eq. (28). To be complete, the authors should also write that the $\sigma_i (i = 1, 2, 3)$ are the Pauli matrices and σ_0 is the identity matrix.

our reply

We defined $\sigma_i (i = 1, 2, 3)$ and σ_0 (in this revised version) after equation (15).

concerning the comment

22) Page 8, Eq. (29). The authors should define this distance, because it never appears before. I believe it is defined by Eq. (17) plus (10) for consistency, but the authors must define it here or before. Moreover, this definition will be useful for giving the explicit expressions of the quantifiers of correlations, T_2 and the others, just in terms of this distance.

our reply

In the line before equation (29), we added: "Using the definition (16)"

concerning the comment

23) Page 8, Eq. (32). Due to the known fact about the closest product state for the Hilbert-Schmidt distance and relative entropy, for Bell-diagonal states the closest product state is expected to be the maximally mixed state, product of the marginals. This should be valid also for the T_2 based on linear relative entropy. The authors should check when their density matrix of Eq. (24) can reduce to a Bell-diagonal state and verify this fact. For instance, it is seen later in Eq. (37) that for $c_1 = c_2$ the density matrix takes the form of a Bell-diagonal state. For this condition, $R_{30} = R_{03} = 0$ and $a_3 = 0$. This means that the closest product state of Eq. (32) is just the maximally mixed density matrix, as expected. The authors should comment on this point.

our reply

After equation (32), we added the following comment:

It is interesting to note that for $c_1 = c_2$, the density matrix ρ (24) becomes a Bell-diagonal state. In this special case, the matrix elements R_{30} and R_{03}, given by (25), vanish and from (31) we have $a_3 = 0$. This implies that the closest product state (32) is just the product of the marginals $\rho_1 = \frac{1}{2}\sigma_0$ and $\rho_2 = \frac{1}{2}\sigma_0$ of ρ.

concerning the comment

24) Page 9, Eq. (33). I do believe that the authors use Eq. (29) to obtain the expression of T_2, together with a_3 given by Eq. (31). They should write like this in the manuscript. Moreover, there is a difference between Eq. (29) and Eq. (33) and it is not clear how the two expressions can be equal. The authors should check this point carefully, since it could be occurred a confusion between the geometric quantifier T_g and the linear entropy based T_2.

our reply

We have checked this point. T_2, given by (33), is obtained by using the equation (17) and (10). T_g, given by (63), is exactly the Hilbert-Schmidt distance (29). We believe that there is no risk of confusion.

concerning the comment

25) Page 10, Figure 1 and the other figures as well. This plot, as well as all the other plots present in the manuscript, is not clear. The authors should indicate in the caption of any figure the values of α corresponding to the solid colored curves. The displayed number size in the axes and the axes labels should be increased. Moreover, in order to make the figures clear also in gray scale, the authors should use different dashing for the curves.

our reply

This is addressed.

concerning the comment

26) Page 15, line 6 from the top. I suggest the authors to refer to L_2^+ as the "quantity" and not as the "linear relative entropy" since it is confusing. Moreover, the authors should comment here about the fact that, when $c_1 = c_2$, that is when the state is reduced to a Bell-diagonal state form, this quantity is zero and the total correlations are equal to the sum $D_2 + C_2 = T_2$, as expected for Bell-diagonal states.

our reply

Before figure 4, we added the following comment:

"Remark that for $c_1 = c_2$, the density matrix ρ (24) has the form of a Bell-diagonal state. In this case, we have $R_{03} = 0$ and $a_3 = 0$ which implies that the quantity L_2^+ is zero. This shows that in Bell-diagonal states, the total correlation T_2 is exactly the sum of quantum discord D_2 and classical correlation C_2."

concerning the comment

27) Page 17, Section 4. At the end of the concluding remarks, the authors should briefly comment on the possibility of ambiguity in the definition of quantum and classical correlations due to degeneracies arising from the optimization procedure over the state space, as pointed out in a very recent paper (F. M. Paula et al., arXiv:1408.1562). For instance, they could write that the possible redefinitions of the quantifiers based on linear relative entropy in order to remove this problem will be treated elsewhere.

our reply

We thank the referee for this interesting reference and suggestion. We added in the conclusion the following comment:

"Finally, we mention the general approach, recently proposed in [50], to remove the unexpected ambiguities leading to multi-valued quantum and classical correlations. Such ambiguities are essentially due to degeneracies arising from the optimization procedures of the distance functions serving as correlations measures. In this context, we believe that it is worthwhile to examine the possible redefinitions of linear relative entropy quantifiers to avoid such problems. We hope to treat this issue in a forthcoming work."

List changes/ Reviewer 2

concerning the comment

In Eq. (17), all the relevant quantifiers are introduced. However, it is not clear enough (at least it is not clearly explained) how difficult is the calculation of the closest states in general, given that in the rest of the manuscript only a very simple case is treated explicitly (for which the original discord could be calculated easily). This point is in my opinion crucial, as the only advantage with respect to

the full relative entropy is probably the analytical simplicity of the measure.

our reply

Indeed equation (17) gives all the relevant quantifiers expressed in terms of linear relative entropy. In this equation T_2, D_2 , C_2 and L_2 are the linearized version of relative entropy-based quantifiers. The explicit derivation of their expressions requires minimization procedures that are in general very complicated to achieve. In this sense, the linear relative offers an alternative way to tackle this issue. On other hand, the states of type (24) are chosen such that solution of the equation (30) can be explicitly derived (we note that for a general X states such solution are extremely difficult to get (see the equation (13) in the paper by Bellomo et al : arXiv:1112.6370 [quant-ph]))

According to the reviewer's comment, we added the following comment (after Eq.(30)): "This cubic equation can be solved using Cardano's formula. It is important to note that for X states without exchange symmetry, the explicit determination of the Bloch coefficients a_i and b_i in (28) is complicated (see [41]). This explains why we deliberately chose to consider only two qubit states of type (24) which are invariant under permutation and parity transformations"

concerning the comment

Another point I'd like to bring to the authors' attention concern Sec. 3.3.1, that is, the calculation of quantum correlations for the X state under study. As stated after Eq. (22) $D_g = D_2$. In this case, I do not see any need to calculate the discord in such a way, as the result should have already be obtained in the literature (probably in Refs. [35,36]). Then, such a detailed section is in my opinion redundant.

our reply

In this subsection, we recall only the main results of geometric discord that are needed for our purpose. In particular, we discuss the conditions on the parameters c_1 and c_2 such that $\lambda_1 \geq \lambda_3$ or $\lambda_3 \geq \lambda_1$. These conditions are important to determine the amount of the other correlations. In this respect, we think that there is no risk of redundancy.