

Title: Pairwise quantum and classical correlations in multi-qubits states via linear relative entropy

## List of changes

We would like to thank the referee for his constructive suggestions and pertinent remarks. We did as best as we could to take into account all his comments.

### concerning the comment

Now, I am not convinced by the physical relevance of such explorations. Indeed, the authors fail to justify on a physical level the linear approximation  $\log \rho \approx \rho - Id$ . Of course, there is a technical justification, since computations become a lot easier. But what happens when one copes with the deep quantum regime?

### our reply

**We thank the referee for this important comment.** The way of introducing the linear entropy (in the first version of the paper) is reformulated. In fact, in the previous version, the reader can understand that the whole analysis are performed only for density matrices approaching completely mixed states. To avoid any misunderstanding, we define (in the revised version) the linear relative entropy (see formula (24)) for arbitrary density matrices. After, equation (29), we mention that for density matrices approaching the unit matrix, the relative entropy coincides with linear relative entropy.

### concerning the comment

There are some strange formulations through the text. For instance, they name “classical state” a diagonal mixed state (e.g., Eq. 19). For the physics community, a classical state is a point in the phase space of a mechanical system. Is it necessary to distort in such confusing way the commonly accepted terminology?

### our reply

We replaced the name “classical states” by “quantum-classical states” which is more appropriate for states presenting zero quantum correlations and containing only classical correlations(classically correlated states). Some authors use the term “classical” to refer to classically correlated states. We agree with the referee that such distortion is not necessary.

### concerning the comment

Another point is the mathematical difference between “pure bipartite states”, which are  $2n \times 2n$  complex matrices and “mixed bipartite states” which are  $4 \times 4$  complex matrices.

### our reply

In the pure bi-partitioning scheme, the whole system is mapped into a two qubit system (this mapping is defined by eqs (6) and (7)) so that the density matrix is of size  $4 \times 4$  (equation (8) gives (10)). The  $4 \times 4$  density matrix for the second scheme is obtained by tracing out  $(n - 2)$  qubits and it is different from the density matrix obtained in the first scheme. These two schemes give all possible bi-partitions of the system.

### concerning the comment

Another strange sentence is found in page 9, below Eq. (41), “The closest classical state to  $\rho_{k,n-k}$  is the eigenvector ...”. Eigenvector of what? The authors should be more careful and rigorous in presenting the mathematical content and notations of their work.

### our reply

1. This sentence is removed and replaced by : ”To find the explicit expression of the closest quantum-classical state to  $\rho_{k,n-k}$ , we follow the procedure reported in [12] for an arbitrary two qubit system.”
2. We added a comment concerning the Schmidt decomposition (8) concerning the definitions of the vectors  $|\mathbf{0}\rangle_k, |\mathbf{1}\rangle_k$  and  $|\mathbf{0}\rangle_{n-k}, |\mathbf{1}\rangle_{n-k}$  (the second line after equation (9)). This responds also to your pervious comment.
3. We also replaced  $a_3$  in equation (46) (and in the sequel of section 5) by  $c_3$  to avoid any confusion with the notation  $a_3$  used in section 4.
4. We replaced the correlation matrix elements  $R_{\alpha,\beta}$  by  $\mathcal{R}_{\alpha,\beta}$  for mixed two qubit states  $\rho_{12}$  to avoid any possible confusion with those associated with the pure case.

### concerning the comment

Also the authors should give a clear account of what is new in their submission with regard to their recent papers that I mentioned above.

### our reply

We agree with the referee that this clarification is of paramount importance for the readers.

Two paragraphs are added to make clear two essentials points (one in the introduction and the second in the beginning of section 3).

In the fourth paragraph in the introduction, we added ”In this picture, the pairwise quantum correlations  $\dots$  existing in  $n$ -qubit systems with parity and exchange invariance.”

In particular, we give a clear account of the novelties of our work which can be summarized as follows:

1. We show that the quantum discord  $D_2$  (with linear relative entropy) is exactly the quantum discord

$D_g$  (based on Hilbert-Schmidt distance) derived in the paper [20] (by two authors of us)

2. In the other hand, we investigate the classification of the states (quantum, quantum-classical, product...), of the system under consideration, according their nature. This question was not considered in the papers [20,21].

3. Finally, we define a linearized version of the relative entropy that is tractable from an analytical point of view in comparison with the unified view proposed in [19].

Similarly, in the beginning of section 3, we stress that the present work is a continuation of the paper [20].

### concerning the comment

There are misprints which should be corrected.

### our reply

They are corrected.